Wheeler High School

To: AP Calculus AB Students

From: Mrs. Barry and Mrs. Quindsland, AP Calculus Instructors

Re: Summer work

Date: May 21, 2010

AP Calculus AB is a college level course covering material traditionally taught in the first semester of college calculus. The course is taught in one semester consisting of daily 90-minute classes. Students need a strong foundation to be ready for the rigorous work required throughout the term. Completing the review packet before the beginning of the course will help ensure a proper background.

This packet consists of a review of topics studied during Algebra II and Analysis or Accelerated Math 1, 2, and 3. Students should expect to work approximately 10 hours on this assignment. The packet should be turned in to Mrs. Barry in Room 520 on the ***first day of school, August 5, 2010*** and will be given a grade based on completeness and accuracy of solutions. **In preparation for the AP test, students need to begin showing all work with logical steps. You must show your work for problems in the review packet. Do not list only an answer.**

Students enrolled in AP Calculus AB will be using a graphing calculator throughout the course since one is required to be used for the AP test. A TI 89 calculator will be available to students for use during the semester if they do not already have one.

Students will have the opportunity to continue studying Calculus for a second term through the BC Calculus course. The BC course will cover material traditionally taught in the second semester of college Calculus. The AB course is a prerequisite for BC Calculus. Students taking both the AB and BC courses and passing the AP Calculus BC test could earn up to 6 hours of college credit depending upon the Advanced Placement policy at the college. Some colleges also award credit for a passing score on the AP Calculus AB exam. Since the AP test is only offered in May, students taking only the AB course during the fall semester should be prepared to study for the test independently, although there will be review sessions available.

The success of each student in the AP Calculus program depends upon diligent effort and practice of newly learned skills. Although a suggested assignment is given for each lesson, completion of some assignments is optional. The previous night's assignment will be reviewed in class each day and there will be ample opportunity to ask questions.

Calculus is a challenging, stimulating, and dynamic field of mathematics and we look forward to sharing our enthusiasm for the material with you. If you have any questions or need to contact someone over the summer, the e-mail contact address is Lynn.Barry@cobbk12.org .

Sincerely,

Mrs. Lynn Barry and Mrs. Rebecca Quindsland
AP Calculus teachers

Calculus Summer Packet

Work the following problems in the space provided. Show all necessary work.

Algebra

Exponents.

1) $\dfrac{\left(8x^3yz\right)^{1/3}(2x)^3}{4x^{1/3}\left(yz^{2/3}\right)^{-1}}$

Factor Completely.

2) $9x^2 + 3x - 3xy - y$

3) $64x^6 - 1$

4) $42x^4 + 35x^2 - 28$

5) $15x^{5/2} - 2x^{3/2} - 24x^{1/2}$

6) $x^{-1} - 3x^{-2} + 2x^{-3}$

7) $e^{2x} + 2 + e^{-2x}$

Rationalize the denominator / numerator.

8) $\dfrac{x}{1-\sqrt{x-2}}$

9) $\dfrac{\sqrt{x+1}+1}{x}$

10) $\dfrac{2-\sqrt{7x}}{2}$

Simplify the rational expression.

11) $\dfrac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

Solve algebraic equations and inequalities.
Use synthetic division to help factor the following, state all factors and roots.

12) $p(x) = x^3 + 4x^2 + x - 6$	13) $p(x) = 6x^3 - 17x^2 - 16x + 7$
14) $p(x) = 25x^3 + 50x^2 - 9x - 18$	15) $p(x) = x^4 - x^3 - 2x - 4$
16) *Explain* why $\frac{3}{2}$ cannot be a root of $4x^5 + cx^3 - dx + 5 = 0$, where c and d are integers.	17)) *Explain* why $x^4 + 7x^2 + x - 5 = 0$ must have a root in the interval [0, 1], ($0 \le x \le 1$)

Solve.

18) $(x + 3)^2 > 4$	19) $\frac{x+5}{x-3} \le 0$	20) $x^3 + 2x^2 - 3x \le 0$
21) $x < \frac{1}{x}$	22) $\frac{x^2-9}{x+1} \ge 0$	23) $\frac{1}{x-1} + \frac{4}{x-6} > 0$
24) $x^2 + 1 > 4x$	25) $\lvert 5x - 1 \rvert > 9$	26) $\frac{1}{\lvert x \rvert} < 1$

27) $\frac{1}{x} < x$	28) $0 < \lvert x-2 \rvert < 8$	29) $x(x-1)(x-2) > 0$
30) $\frac{x^2-9}{x+1} > 0$	31) $\lvert x \rvert < 2$	32) $\lvert 2x+1 \rvert < \frac{1}{4}$

Solve the system.

33) $x - y + 1 = 0$ $y - x^2 = -5$	34) $x^2 - 4x + 3 = y$ $-x^2 + 6x - 9 = y$

Graphing and Functions

Linear Graphs - *Write the equation of the line described below.*

35) Passes through the point (2, -1) and has slope -1/3.	36) Passes through the point (4, -3) and is perpendicular to $3x + 2y = 4$.	37) Passes through (-1, -2) and Is parallel to $y = \frac{3}{5}x - 1$.

Conic Sections - *Write the equation in standard form and identify the conic.*

38) x = 4y ^{2} + 8y - 3	39) $x^2 + y^2 - 4x + 2y - 4 = 0$

40) $4x^2 - 16x + 3y^2 + 24y + 52 = 0$	41) $x^2 - 4y^2 + 2x + 24y - 19 = 0$

Functions

Find the domain and range of the following.
domain restrictions - denominator ≠ 0, argument of a log or ln > 0, radicand of even index must be ≥ 0
range restrictions- reasoning, if all else fails, use graphing calculator

42) y = $\frac{3}{x-2}$	43) y = log(x - 3)
44) y = $x^4 + x^2 + 2$	45) y = $\sqrt{2x-3}$
46) y = | x - 5 |	47) $y = \frac{\sqrt{x+1}}{x^2-1}$ (domain only)
48) $f(x) = \frac{x-3}{x^2+3x+2}$	49) Given f(x) below, graph over the domain [-3, 3]. What is the range? $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 1 & \text{if } -1 \leq x < 0 \\ x-2 & \text{if } x < -1 \end{cases}$

Compositions and Inverses - *Find the compositions and inverses as indicated below.*

Let: f(x) = x^2 + 3x - 2 g(x) = 4x - 3 h(x) = ln*x* w(x) = $\sqrt{\mathbf{x-4}}$

50) g $^{-1}$(x)	51) h $^{-1}$(x)	52) w $^{-1}$(x), for x ≥ 4	53) f(g(x))

54) h(g(f(1)))	55) g(h(x))	56) g(w(x))	

57) Does $y = 3x^2 - 9$ have an inverse function? Explain your answer

Let: f(x) = 2x, g(x) = -x and h(x) = 4

58) $(f \circ g)(x)$	59) $(h \circ f)(x)$	60) $(g \circ h \circ g)(x)$

Basic Shapes of Curves:

Sketch the graphs on separate paper. You may use your graphing calculator to verify the graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

61) $y = \sqrt{x}$	62) $y = \ln x$	63) $y = \lvert x - 2 \rvert$
64) $y = \frac{1}{x}$	65) $y = \frac{1}{x-2}$	66) $y = \frac{x}{x^2-4}$
67) $y = 2^{-x}$	68) $y = 3 \sin 2\left(x - \frac{\pi}{6}\right)$	69) $f(x) = \begin{cases} \sqrt{25-x^2} & \text{if } x < 0 \\ \frac{x^2-25}{x-5} & \text{if } x \geq 0,\ x \neq 5 \\ 0 & \text{if } x = 5 \end{cases}$
70) $y = 3^x$	71) $x^2 + y^2 - 4x - 32 = 0$	72) $y = e^x$

Asymptotes – Identify any asymptotes.

73) $y = \frac{1}{x-1}$	74) $x^2 - y^2 = 1$	75) $y = \frac{x^2}{x^2-1}$
76) $y = \frac{3x^2}{2x^2-3x+3}$	77) $y = \frac{5x^2-5x+1}{x-1}$	78) $y = \frac{2x^2}{3x^3-4x+1}$

Even and Odd Functions - *Identify as odd, even, or neither. Show substitutions!*

Even: f (x) = f (-x) Odd: f (-x) = - f (x)

79) f(x) = $x^3 + 3x$	80) *f(x)* = $x^4 - 6x^2 + 3$	81) $f(x) = \frac{x^3 - x}{x^2}$	82) f(x) = sin 2x
83) f(x) = $x^2 + 1$	84) f(x) = $x(x^2 - 1)$	85) f(x) = $\frac{1+\lvert x \rvert}{x^2}$	86) $f(x) = \cos^2 x$
87) What type of function results from the product of two even functions? odd functions?			

Test for symmetry. Show substitutions.

Symmetric to y axis: replace x with - x and relation remains the same.

Symmetric to x axis: replace y with - y and relation remains the same.

Origin symmetry: replace x with - x, y with - y and the relation is equivalent.

88) $y = x^4 + x^2$	89) y = sin(x)	90) y = cos(*x*)
91) $y = \frac{-x}{x^5 + x^3}$	92) y = $\frac{\lvert x \rvert}{x^2 + 1}$	93) $y = \frac{1}{x - 2}$
94) x = $y^2 + 1$	95) $y = x^3$	96) $y = x^5 - 3x^3 + x$

LOGARITHMIC AND EXPONENTIAL FUNCTIONS

Simplify Expressions.

97) $\log_4\left(\frac{1}{16}\right)$	98) $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3\left(\frac{1}{27}\right)$	99) $\log_9 27$	100) $\log_{125}\left(\frac{1}{5}\right)$
101) $\log_w w^{45}$	102) $\ln e$	103) $\ln 1$	104) $\ln e^2$

Solve equations.

105) $\log_6(x + 3) + \log_6(x + 4) = 1$	106) $\log x^2 - \log 100 = \log 1$	107) $3^{x+1} = 15$

Find the inverse.

108) $f(x) = 7^x$	109) $f(x) = \log_3 x$	110) $f(x) = \ln(x + 5)$

TRIGONOMETRY

Unit Circle – find the following without using a calculator.

111) $\sec\left(-\frac{\pi}{6}\right)$	112) $\tan\left(\frac{9\pi}{4}\right)$	113) $\cos\left(\frac{11\pi}{3}\right)$	114) $\sin\left(\frac{11\pi}{4}\right)$
115) $\cot 8\pi$	116) $\tan\left(\frac{5\pi}{2}\right)$	117) $\csc\left(\frac{-5\pi}{6}\right)$	118) $\sin\left(\frac{7\pi}{3}\right)$

State the domain, range, and fundamental period for each function.

119) $y = \sin x$	120) $y = \cos x$	121) $y = \tan x$

Identities – simplify.

122) $\dfrac{(\tan^2 x)(\csc^2 x)-1}{(\csc x)(\tan^2 x)(\sin x)}$	123) $1 - \cos^2 x$	124) $\sec^2 x - \tan^2 x$

Solve equations.

125) $\cos^2 x = \cos x + 2$ $\quad 0 \le x \le 2\pi$	126) $2\sin(2x) = \sqrt{3}$ $\quad 0 \le x \le 2\pi$
127) $4\cos^2 x = 1$ $\quad \lvert x \rvert \le \pi$	128) $\cos^2 x + \sin x + 1 = 0$ $\quad -\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$
129) $4\sin x \cos x = \sqrt{3}$ $\quad 0 \le x \le 2\pi$	130) $2\sin^2 x = 1$ $\quad 0 < x < 2\pi$

Inverse Trig Functions – Evaluate.

131) $\text{Arcsin}(1)$	132) $\text{Arcsin}\left(-\dfrac{\sqrt{2}}{2}\right)$	133) $\text{Arccos}\left(\dfrac{\sqrt{3}}{2}\right)$	134) $\text{in}\left(\text{Arccos}\left(\dfrac{\sqrt{3}}{2}\right)\right)$

135) State domain and range for:

a) Arcsin(x)

b) Arccos (x)

c) Arctan (x)

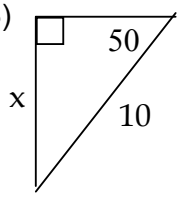
Graphing – State the amplitude and period of the following functions.

136) $y = -2\sin(2x)$	137) $y = 1/4\cos(4x + 12)$

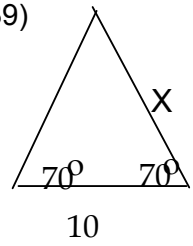
Geometry

Right Triangle Trig - Find the value of x.

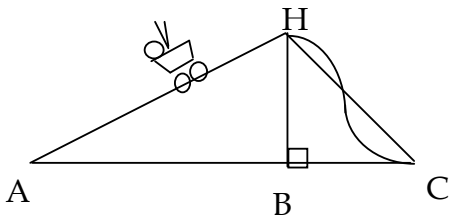
138)



139)



140)



The roller coaster car shown in the diagram above takes 23.5 sec. to go up the 23 degree incline segment AH and only 2.8 seconds to go down the drop from H to C. The car covers horizontal distances of 180 feet on the incline and 60 feet on the drop.

a) How high is the roller coaster above point B?
b) Find the distances AH and HC.
c) How fast (in ft/sec) does the car go up the incline?
d) What is the approximate average speed of the car as it goes down the drop? (Assume the car travels along HC.)
e) Is your approximate answer too big or too small?

(Advanced Mathematics, Richard G. Brown, Houghton Mifflin,1994, pg 336)